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Squeeze film effect at elastohydrodynamic lubrication of plain journal bearings

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ABSTRACT

The aim of the present work is to study the influence of the elastic displacements of bearing surface coatings on the hydrodynamic lubrication of dynamically loaded journal bearing with finite length. The problem is investigated for a Newtonian lubricant under isothermal and isoviscous conditions. The bearing sleeve is covered with a thin elastic coating whose radial distortions are of the same order of magnitude as the film thickness. The elasticity part of the problem is studied in conformity with the Vlassov model of an elastic foundation and comparisons with other models are carried out. The numerical solution is done by finite difference method with successive over-relaxation procedure. The presented results are obtained for prescribed loci of the shaft centre which corresponds of squeeze film effect.

Keywords: elastohydrodynamic (EHD) lubrication, squeeze film effect, journal bearing.

INTRODUCTION

Nowadays, the need for higher speed and at the same time reliable operation of various types of rotating machinery continues to grow. An important factor in achieving this goal is the ability to accurately predict the dynamic response and stability of the rotor-bearing system [1].

Each rotating machine supported by one or more bearings, whose role is very important in the system as a whole, because namely the bearings are the components that allow relative movement between the stationary and moving parts. As it is well known, there are two main types of bearings which are normally used in applications of the rotor-bearing systems. These are different types of journal bearings and rolling-elements bearings.

Journal bearings are used widely and successfully for thousands of years and may be they are the most used machine element in our civilization [1]. Currently large numbers and various types of sliding bearings are used in countless applications such as small electric motors, hard disk drives, micro-electro-mechanical systems, automotive and aircraft piston engines, large steam turbines for electric power generation, etc.

For dynamically loaded journal bearings are typical the squeeze film and dynamic film effects which play significant role in the dynamic behaviour of the rotor. Since the thin oil film that separates the moving surfaces supports the rotor load, it acts as a spring and provides a damping effect due to the squeeze film effect.

Investigations in this direction have been made

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by many authors in recent decades [2-7]. The importance of the problem implies a phenomenon studying in combination with other factors influencing the lubrication process of the hydrodynamic (HD) journal bearings.

It is well-known that elastic deformation of the material of the contact bearing surfaces, induced by the HD fluid pressure, can significantly change the fluid film profile, modify the pressure distribution and, therefore, alter the performance characteristics of journal bearings [8]. The use of surface coatings (on bush or/and on journal) such as white metals and various elastomers can lead to significant deformations of the coated surface which can be about the size of thickness of lubricating film. These coatings, used with the aim of reducing wear, are generally characterized by low values of the modulus of elasticity [9].

The effect of the deformation of the bearing bush on the journal bearing performance characteristics was reported by many investigators [8-13]. Many of these papers are related to infinitely long/short bearings and as whole the comparison between different formulas for elastic deformations calculations is absent.

In this work, we are interested in the study of the influence of the elastic displacements of bearing surface coatings on the HD lubrication of dynamically loaded journal bearing with finite length. The problem is investigated for a Newtonian lubricant under isothermal and isoviscous conditions. The elasticity part of the problem is studied in conformity with the Vlassov model of an elastic foundation and comparisons with other models are carried out. The numerical solution is done by finite difference method with over-relaxation procedure; such as the presented results are concerned toprescribed loci of the shaft centre resulting from the squeeze film effect.

ELASTOHYDRODYNAMIC MODEL

The cross section of a 360 degree plain journal bearing is presented in a Fig. 1. The main components of the considered tribological system, used in the hydrodynamic theory of lubrication, are the bearing shaft (journal), the lubricant and the bearing bushing which is plain cylindrical sleeve wrapped around the journal.

The bearing bush is rigidly supported while the journal moves and in our case it is dynamically loaded in a radial direction. About the journal motion it will be assumed squeeze film effect, i.e. a vibration velocity de/dt of the shaft centre in a direction of the centre's line C_L (Fig.1). Its frequency is equal to the angular velocity of the shaft, which is considered to be constant and equal to ω. The amplitude of vibration will be about to the half of the radial clearance c. Hence $e = e_0 + 0,6c \sin \omega t$, as $e_0 = 0,2c$.

Both of the radiuses (of bush and of journal) are approximately equal because of which it is possible to neglect the curve shape of the fluid film. An elastic liner with elastic properties μ and E (Poisson's ratio and Young's modulus, respectively) is press-fitted in a rigid housing. The liner thickness d is assumed to be of the same order of magnitude as the lubricant thickness h. Other assumptions for the present analysis are: the journal is rigid; the lubricant with Newtonian properties is incompressible fluid under isoviscous and isothermal conditions; the flow is laminar and the inertia effects are negligible.

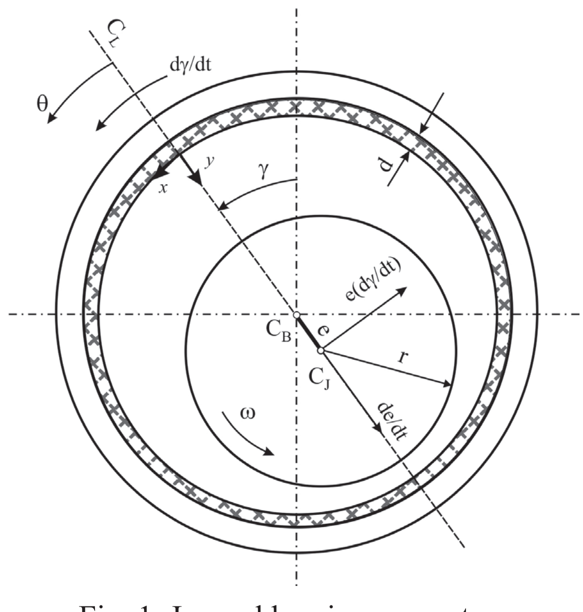

Fig. 1. Journal bearing geometry.

The governing equations for the elastohydrodynamic (EHD) lubrication of a dynamically loaded plain journal bearing with finite length are: the equation for HD pressure distribution (Reynolds equation), equation for film thickness geometry and elasticity equation (for estimation of the elastic distortions of bearing liner).

Derivation of the Reynolds equation

Our interest in this section is a derivation of an equation for the pressure field within a dynamically loaded bearing.

At neglecting the body and inertia forces the Navier-Stokes equations, governing the motion in the lubricant, for incompressible viscose lubricant flow are:

$$0 = -\nabla p + \eta \nabla^2 V \, , \qquad (1)$$

where p is the hydrodynamic pressure, η is the lubricant' dynamic viscosity, V is the fluid velocity.

The flow field model is completed by the incompressible fluid continuity equation

$$\nabla V = 0 \qquad (2)$$

It is well known that one of the main assumptions for the journal bearings lubrication theory is that the fluid film is thin compared with the journal radius and the curvature of the film is neglected. Then the field equations (1) and (2) given in Cartesian coordinates can be reduced to the following:

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\eta}\frac{\partial p}{\partial x} \qquad (3.a)$$

$$0 = \frac{1}{\eta}\frac{\partial p}{\partial y} \qquad (3.b)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\eta}\frac{\partial p}{\partial z} \qquad (3.c)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0 \qquad (3.d)$$

where u, v, w are the velocity components on the Cartesian rectangular coordinates x, y, z.

The boundary conditions for the velocities components are at $y = 0$:

$$u = 0 \quad v = 0 \quad w = 0 \qquad (4)$$

at $y = h$

$$u = u_h = \omega r + \frac{de}{dt}\sin\theta - e\frac{d\gamma}{dt}\cos\theta \qquad (5.a)$$

$$v = v_h = \omega r\frac{\partial h}{\partial x} + \frac{de}{dt}\cos\theta + e\frac{d\gamma}{dt}\sin\theta \qquad (5.b)$$

$$w = w_h = 0 \qquad (5.c)$$

Here h is the fluid film thickness, ω is the shaft angular velocity, r is radius of the journal, e is the bearing eccentricity, $\theta = x/r$ is a circumferential coordinate, γ represents attitude angle, t is the time.

Integrating Eqns. (3.a) and (3.c) with respect to y and employing the above boundary conditions, for the velocity components are obtained:

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x}y^2 + C_1(x,z,t)\,y + C_2(x,z,t) \qquad (6)$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z}y^2 + C_3(x,z,t)\,y + C_4(x,z,t) \qquad (7)$$

where C_1, C_2 and C_3, C_4 are integration constants.

At satisfying the boundary conditions (4) and (5) the integration constants are determined as follows:

$$C_1(x,z,t) = \frac{u_h}{h} - \frac{1}{2\eta}\frac{\partial p}{\partial x}h \qquad C_2(x,z,t) = 0 \quad (8)$$

$$C_3(x,z,t) = -\frac{1}{2\eta}\frac{\partial p}{\partial z}h \qquad C_4(x,z,t) = 0 \quad (9)$$

After substitution of (8) and (9) in (6) and (7) respectively, the expressions for the velocity components in direction of coordinate axes x and

z are received in the form:

$$u(x,z,t)=\frac{1}{2\eta}\frac{\partial p}{\partial x}(y-h)y+\frac{u_h}{h}y \tag{10}$$

$$w(x,z,t)=\frac{1}{2\eta}\frac{\partial p}{\partial z}(y-h)y \tag{11}$$

u is the sum of the flow due to circumferential pressure gradient $\partial p/\partial x$ and the flow due to the no-slip boundary conditions. Note further that the pressure gradients $\partial p/\partial x$ and $\partial p/\partial z$ are independent of y.

For the considered case it is assumed a linear distribution of the radial velocity $v(x,y,z,t)$ using the formulation in [14] and [4]:

$$v=C_5(x,z,t)y+C_6(x,z,t) \tag{12}$$

At satisfying the boundary conditions (4) and (5) the integration constants are obtained as follows:

$$C_5(x,z,t)=\frac{v_h}{h}=$$
$$=\frac{1}{h}\left(\omega r\frac{\partial h}{\partial x}+\frac{de}{dt}\cos\theta+e\frac{d\gamma}{dt}\sin\theta\right) \tag{13.a}$$

$$C_6(x,z,t)=0 \tag{13.b}$$

Than the final form for this velocity component is given by:

$$v(x,z,t)=\frac{v_h}{h}y=$$
$$=\frac{1}{h}\left(\omega r\frac{\partial h}{\partial x}+\frac{de}{dt}\cos\theta+e\frac{d\gamma}{dt}\sin\theta\right)y \tag{14}$$

After integration of continuity equation (3.d) with respect to y from $y=0$ to $y=h$

$$\int_0^h\frac{\partial u}{\partial x}dy+\int_0^h\frac{\partial v}{\partial y}dy+\int_0^h\frac{\partial w}{\partial z}dy=0 \tag{15}$$

it is received

$$\int_0^h\frac{\partial u}{\partial x}dy+v_h+\int_0^h\frac{\partial w}{\partial z}dy=0 \tag{16}$$

Applying Leibniz's rule for the differentiationunder the integral sign,

$$\int_0^{h(x)}\frac{\partial}{\partial x}\left[f(y,x)\right]dy=$$
$$=\frac{\partial}{\partial x}\int_0^{h(x)}f(y,x)dy-f(h,x)\frac{dh}{dx}$$

the first and the third terms on the left side of Eq. (16) are transformed to:

$$\int_0^h\frac{\partial u}{\partial x}dy=\frac{\partial}{\partial x}\int_0^h udy-u_h\frac{\partial h}{\partial x} \tag{17}$$

$$\int_0^h\frac{\partial w}{\partial z}dy=\frac{\partial}{\partial z}\int_0^h wdy-w_h\frac{\partial h}{\partial z} \tag{18}$$

Since $w_h=0$ from the boundary conditions (5.c) it follows that (18) becomes:

$$\int_0^h\frac{\partial w}{\partial z}dy=\frac{\partial}{\partial z}\int_0^h wdy \tag{19}$$

Substitution of (17) and (19) in (16) y ields

$$\frac{\partial}{\partial x}\int_0^h udy-u_h\frac{\partial h}{\partial x}+v_h+\frac{\partial}{\partial z}\int_o^h wdy=0 \tag{20}$$

In the considered case of unsteady state motion of the lubricant the following equals are valid [4]:

$$\frac{\partial h}{\partial t}=\frac{de}{dt}\cos\theta+e\frac{d\gamma}{dt}\sin\theta \tag{21.a}$$

$$-u_h\frac{\partial h}{\partial x}+v_h=\frac{de}{dt}\cos\theta+e\frac{d\gamma}{dt}\sin\theta \tag{21.b}$$

Considering (21.a) and (21.b) follows:

$$\frac{\partial h}{\partial t}=-u_h\frac{\partial h}{\partial x}+v_h \tag{21.c}$$

Taking into account (21.c) the above equation (20) takes a form:

$$\frac{\partial}{\partial x}\int_0^h udy+\frac{\partial h}{\partial t}+\frac{\partial}{\partial z}\int_o^h wdy=0 \tag{22}$$

At introducing the flow parameters q_x, q_z which are presented respectively as

$$q_x=\int_0^h udy \tag{23}$$

$$q_z=\int_0^h wdy \tag{24}$$

the Eq. (22) takes a form:

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_z}{\partial z}+\frac{\partial h}{\partial t}=0 \tag{25}$$

The expressions of the velocity components (10) and (11) can be placed respectively in integrand functions of (23) and (24); such the flow parameters become:

$$q_x=\int_0^h udy=\frac{1}{6\eta}\frac{\partial p}{\partial x}h^3-\frac{1}{4\eta}\frac{\partial p}{\partial x}h^3+$$

$$+\frac{1}{2}u_h h=-\frac{h^3}{12\eta}\frac{\partial p}{\partial x}+\frac{h}{2}u_h \tag{26}$$

$$q_z=\int_0^h wdy=\frac{1}{6\eta}\frac{\partial p}{\partial z}h^3-\frac{1}{4\eta}\frac{\partial p}{\partial z}h^3=$$

$$=-\frac{h^3}{12\eta}\frac{\partial p}{\partial z} \tag{27}$$

The derivatives of flow parameters with respect to axes x and z are obtained at rendering into account (26) and (27):

$$\frac{\partial q_x}{\partial x}=-\frac{1}{12\eta}\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{1}{2}\frac{\partial}{\partial x}\left(hu_h\right) \tag{28}$$

$$\frac{\partial q_z}{\partial z}=-\frac{1}{12\eta}\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) \tag{29}$$

The obtained expressions (28) and (29) subsequently are substituted in (25), which is representing as:

$$\frac{1}{12\eta}\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{1}{12\eta}\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=$$

$$=\frac{1}{2}\frac{\partial}{\partial x}\left(hu_h\right)+\frac{\partial h}{\partial t} \tag{30}$$

For the first term on the right-hand side of above equation can be written [4]

$$\frac{\partial}{\partial x}\left(hu_h\right)=u_h\frac{\partial h}{\partial x}+h\frac{\partial u_h}{\partial x}\approx\omega R\frac{\partial h}{\partial x} \tag{31}$$

Then the Eq. (30) represents as

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial z}\right)=$$

$$=6\omega r\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t} \tag{32}$$

In the literature [3 - 5] an equation from the type of (32) is known as Reynolds type equation for pressure field within a bearing or for hydrodynamic pressure distribution in the lubricant film of finite length journal bearing under unsteady state conditions. This equation differs from the classical Reynolds equation in the last term on the right-hand side only. It takes into account the pressure changes due to the temporal variation of the film thickness.

Film thickness geometry and elasticity equation

The approach used in the present study aims to superimpose the deformation of the layer on the bush (the other components of the bearing and the journal are treated as rigid), caused by generated HD pressure, onto the oil film thickness. The gap thickness is then modified in order to be account for the estimated elastic deformation as follows:

$$h = c + e\cos\theta + \delta \tag{33}$$

where c is a radial bearing clearance and δ is the surface points radial displacement of the bush liner.

The last term of this equation takes into account the influence of the elastic layer deformations. In the current paper the liner's surface points radial displacements are determined according to the one of more precise models, namely the three-dimensional model of elastic foundation suggested in [15]. The solution of the elasticity part of the problem by Vlassov but for the case of thin layer is worked up in details in [16], because of which only the final form is given here, namely

$$\delta = \frac{(1-2\mu)(1-\mu^2)d}{E(1-\mu)^2}p \tag{34}$$

where d is a thickness of the bearing liner, μ is the Poisson's ratio, E is the Young's modulus.

Furthermore it will be made a comparison of the results for elastic deformations calculated by other formulas: one of them by Higginson [8, 17]

$$\delta = \left(1 - \frac{2\mu^2}{1-\mu}\right)\frac{d}{E}p \tag{35}$$

and the other by Kodnir (Winkler-Zimmerman hypothesis) [18, 19]

$$\delta = \frac{2(1-\mu^2)}{\pi}\frac{d}{E}p \tag{36}$$

Load carrying capacity

The resultant force W, which is balanced by the load applied to the shaft, is given by:

$$W = \sqrt{W_1^2 + W_2^2} \tag{37}$$

where W_1 and W_2 are the total components in radial (along the line of centers) and tangential directions respectively.

The Sommerfeld number, which represents a coefficient of load capacity, may be express quantitative as

$$S = \frac{W\beta^2}{\eta\omega r L} \tag{38}$$

L is the bearing axial length while $\beta = c/r$ is the clearance ratio.

SOLUTION PROCEDURE

Non-dimensional form of equations

The solution of Eq. (32) will be done by numerical methods. By reason of this and also for keeping the conditions of geometric and kinematic similarity at the solution, this equation must be represented in a non-dimensional form

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{c^3H^3}{\eta}\frac{\partial\Pi}{\partial\theta}\frac{6\eta\omega r^2}{rc^2}\right)+$$
$$\frac{1}{L/2}\frac{\partial}{\partial z_1}\left(\frac{c^3H^3}{\eta}\frac{\partial\Pi}{\partial z_1}\frac{6\eta\omega r^2}{(L/2)c^2}\right)=$$
$$6\omega r\frac{c}{r}\frac{\partial H}{\partial\theta}+12\frac{c\omega}{2}\frac{\partial H}{\partial\tau} \tag{39}$$

by introducing the following dimensionless variables: $\Pi = p.(c/r)^2/6\eta\omega$ - pressure, $H = h/c$ - film thickness, $\tau = t.\omega/2$ - time, $z_1 = z/(L/2)$ - axial coordinate

After transformations the dimensionless form of the equation for HD pressure distribution becomes:

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial\Pi}{\partial\theta}\right)+\alpha\frac{\partial}{\partial z_1}\left(H^3\frac{\partial\Pi}{\partial z_1}\right)=$$
$$\frac{\partial H}{\partial\theta}+\frac{\partial H}{\partial\tau} \tag{40}$$

where $\alpha = 2r / L$ - diameter to length ratio and

$$\frac{\partial H}{\partial \theta} = -\varepsilon \sin\theta$$

$$\frac{\partial H}{\partial \tau} = \frac{\partial \varepsilon}{\partial \tau}\cos\theta + \varepsilon\frac{\partial \gamma}{\partial \tau}\sin\theta \tag{41}$$

The Eq. (33) for film thickness can be rendered dimensionless through the relevant substitutions and written as:

$$H = 1 + \varepsilon\cos\theta + \bar{\delta} \tag{42}$$

Here $\varepsilon = e/c$ is the eccentricity ratio and $\bar{\delta}$ are dimensionless displacements.

Furthermore the radial distortions of the bearings liner surface points /Eq. (34)/ are presented in non-dimensional form as:

$$\bar{\delta} = \frac{6\eta\omega r^2}{c^3}\frac{(1-2\mu)(1-\mu^2)d}{E(1-\mu)^2}\Pi \tag{43}$$

The hydrodynamic film forces acting on the system are expressed in terms of dimensionless quantities by

$$\bar{W}_1 = -\int_{-1}^{1}\int_{0}^{2\pi}\Pi\cos\theta\, d\theta dz_1$$

$$\bar{W}_2 = \int_{-1}^{1}\int_{0}^{2\pi}\Pi\sin\theta\, d\theta dz_1 \tag{44}$$

where $\bar{W}_1$ and $\bar{W}_2$ are radial and tangential components.

Then the resultant dimensionless load carrying capacity $\bar{W}$ can be calculated by

$$\bar{W} = \sqrt{\bar{W}_1^2 + \bar{W}_2^2} = \frac{\beta^2}{6\eta\omega rL}W \tag{45}$$

The Sommerfeld number in present analysis is equal to:

$$S = 6\bar{W} \tag{46}$$

Numerical solution

The computation is done using the finite difference method, as the numerical procedure is iterative and the successive over-relaxation method is used to improve the convergence rate [4, 12, 16]. About the pressure are used the Reynolds boundary conditions; such in the circumferential direction it is assumed that the positive pressure terminates at θ^* where the pressure gradient angle is zero.

Prior to starting the iteration process, the film pressure is initially determined considering the bearing liner as rigid one. With the help of this pressure the non-dimensional deformation is computed. With this deformation the Reynolds equation (40) is solved again to get a new film pressure distribution. This process is repeated through iteration technique till convergence is achieved. With the help of this film pressure after convergence, the bearing performance characteristics are obtained.

NUMERICAL RESULTS AND DISCUSSION

The presented results are obtained at different elasticity parameters ($\delta \neq 0$). The later are related to four separate cases: $E = 2.10^{11}$ Pa, $\mu = 0,25$ (steel - rigid case); $E = 1,63.10^8$ Pa, $\mu = 0,38$ (elastomer - soft case 1); $E = 7,33.10^7$ Pa, $\mu = 0,4$ (elastomer - soft case 2); $E = 4,07.10^7$ Pa, $\mu = 0,41$ (elastomer - soft case 3).

In Figs. 2 and 3 are given the longitudinal centre-line distributions of the HD pressure, film thickness and radial displacements of the bearing liner surface points. As it was expected, for the soft cases a significant reduction of the maximum HD pressure is observed while for the radial displacements the effect is reversed. This influence is a more visible for the softer materials of the elastic liner. Under deformability conditions the fluid film geometry is changed in the bottom; such a increase of H in the middle section is observed.

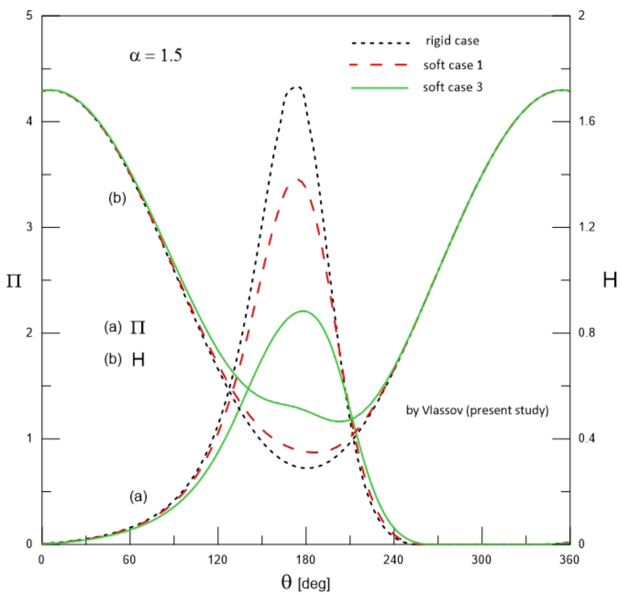

Fig. 2. Distribution of HD pressure Π and film thickness H under different elasticity conditions.

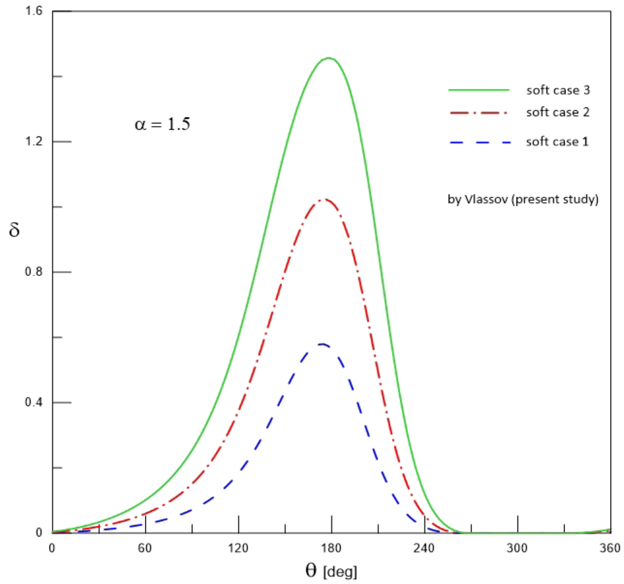

Fig. 3. Displacements distribution at the bearing centre line.

The results in Figs. 4 and 5 about time variation of HD pressure (caused by the vibration velocity de/dt of the shaft centre - squeeze film effect) show the same tendency for reduction of maximal values of Π as above. The graphics presented in a Fig. 5 give results for pressure obtained by different formulas for elastic deformations calculations.

The influence of the elasticity layer deformability on the Sommerfeld number is displayed in a Fig. 6. Analogically as the pressure the S values becomes smaller for the soft case; such this tendency can be recognized more clear for the maximal values of the coefficient of load capacity.

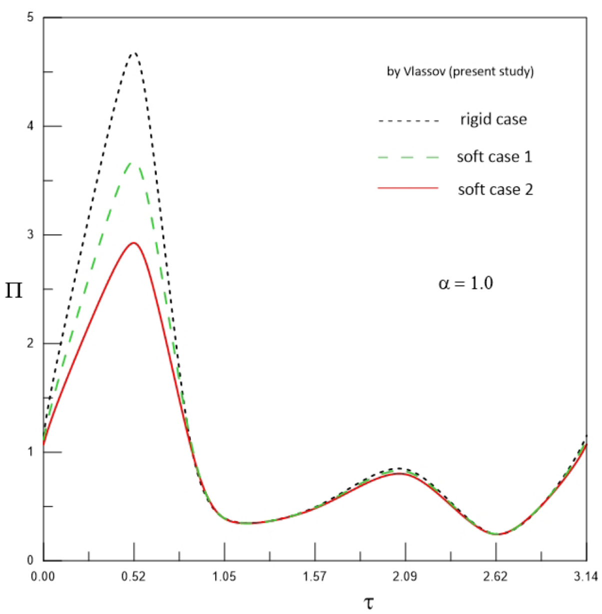

Fig. 4. Time variation of Pwith radial velocity under different elasticity conditions.

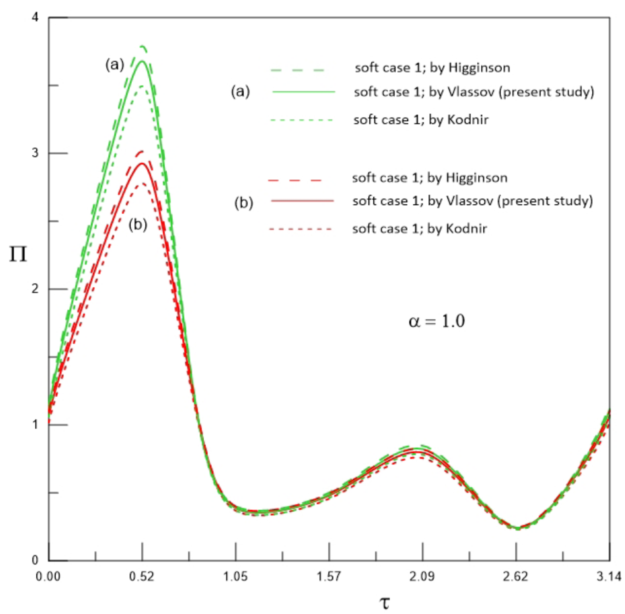

Fig. 5. HD pressure versus time – calculated by different formulas for deformations.

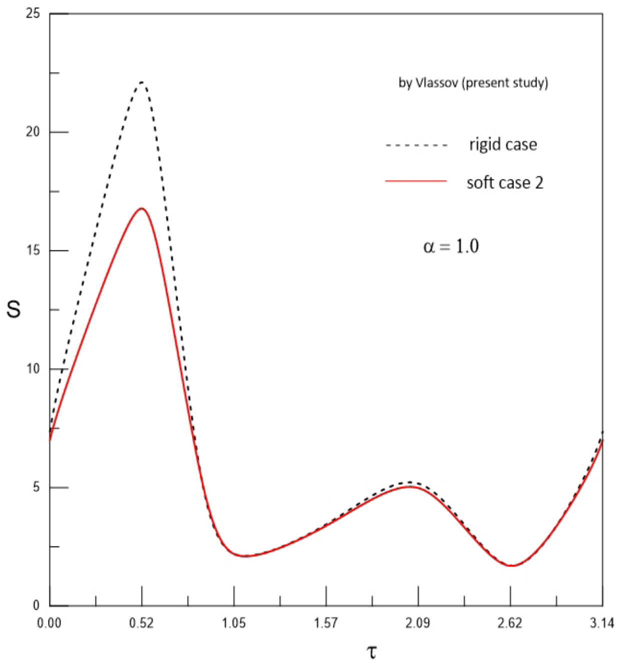

Fig. 6. Variation of Sommerfeld number with time under different deformability conditions.

CONCLUSIONS

On the base of Navier –Stokes and continuity equations the generalized Reynolds equation for lubrication of dynamically loaded journal bearing with finite length is derived in details.

The numerical results for squeezed film conditions show that the maximum pressure for elastic bearing liner is smaller than that for bearing with rigid layer, and this effect is more significant for the softer liner materials. A similar trend was observed by other investigators for steady-state or transient problems of journal bearings. Deformability of the layer modifies and film shape; such appreciable increase in minimum film thickness for more flexible bearing liner is observed and it is also a favorable effect from the designer's viewpoint.

The differences between the results obtained by different formulas for elastic deformations calculations are not so high (till 8%) and all these methods are applicable and have own significance.

REFERENCES

1. U.Yucel, Calculation of Dynamic Coefficients for Fluid Film Journal Bearings, Journal of Eng. Sciences, 11, (3), 2005, 335-343.
2. J. Lin, Squeeze Film Characteristics of Finite Journal Bearings: Couple Stress Fluid Model, Tribology International, 31, 4, 1998, 201-207.
3. A. Szeri, Fluid film lubrication, Cambridge Univ. Press, 1st Ed., Cambridge, U.K., 1998.
4. J. Javorova, Ph.D. Thesis, One solution of the non-stationary problem of the elastohydrodynamic theory of lubrication, University of Transport, Sofia, 1998, (in Bulgarian).
5. B. Hamrock, S. Schmid, B. Jacobson, Fundamentals of Fluid Film Lubrication, 2nd Ed., Marcel Dekker Inc, N.Y., 2004.
6. Z.Guo, T. Hirano, R. Kirk, Application of CFD Analysis for Rotating Machinery - Part I: Hydrodynamic, Hydrostatic Bearings and Squeeze Film Damper, J. Eng. Gas Turbines Power, 127, 2, 2005, 445-451.
7. G.J. Reddy, C.E. Reddy, K.R. Prasad, Effect of viscosity variation on the squeeze film performance of a narrow hydrodynamic journal bearing operating with couple stress fluids, Proc. of Inst. of Mech.Eng, Part J, J. of Eng, Tribology, 222, 2, 2008, 141-150.
8. G. Higginson, The theoretical effects of elastic deformation of the bearing liner, Proc. Mech. Eng., vol.180, 1966, 31-38.
9. L. Dammak, E. Hadj-Taieb, Finite Element Analysis of Elastohydrodynamic Cylindrical Journal Bearing, Fluid Dyn. Mater. Process, 6, (4), 2010, 419-430.
10. T. Osman, Effect of lubricant non-Newtonian behaviour and elastic deformation on the dynamic performance of finite journal plastic bearings, Tribology Letters, 17, (1), 2004, 31-40.
11. Y. Ma, Performance of dynamically loaded journal bearings lubricated with couple stress fluids considering the elasticity of the liner, J Zhejiang Univ Sci A, 9, (7), 2008, 916-921.

12. H. Attia, S. Bouaziz, M. Maatar, T. Fakhfakh, M. Haddar, Hydrodynamic and elastohydrodynamic studies of a cylindrical journal bearing, Journal of Hydrodynamics, 22, (2), 2010, 155-163.
13. S.Guha, On the Steady-State Performance of HD Flexible JB of Finite Width Lubricated by Ferro Fluids with Micro-Polar Effect, Int. J. Mech. Eng. & Rob. Res., 1, (2), 2012, 32-49.
14. A. Kelzon, J. Tcimanskii, V. Jakovlev, Dynamic of rotors in elastic bearings, Nauka, Moscow, 1982,(in Russian).
15. V. Vlassov, U. Leontiev, Beams, plates and shells on elastic foundation, Moscow, 1960, (in Russian).
16. J. Javorova, V. Alexandrov, K. Stanulov, Static and dynamic performance of EHD journal bearings in turbulent flow, Proc. of the 3rd Intern. Sci. Conf. "Power Transmissions' 09", Thessaloniki, Greece, 2009, 453-460.
17. D. Dowson, G. Higginson, Elastohydrodynamic Lubrication, 2nd Ed., Pergamon Press, Oxford, 1977.
18. D. Kodnir, Contact hydrodynamic of machine details lubrication, Moscow, 1976, (in Russian).
19. V. Tishchenko, Models of layer deformation of an elastic half space, Elsevier Journal of Soviet Mathematics, 65, (1), 1993, 1396-1402.